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2007 J. Phys.: Condens. Matter 19 056203

(http://iopscience.iop.org/0953-8984/19/5/056203)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **19** (2007) 056203 (10pp) doi:10.1088/0953-8984/19/5/056203

Optically controlled single-qubit rotations in self-assembled InAs quantum dots

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Received 24 August 2006, in final form 18 December 2006
Published 17 January 2007
Online at stacks.iop.org/JPhysCM/19/056203

Abstract
We present a theory of the optical control of the spin of an electron in an InAs quantum dot. We show how two Raman-detuned laser pulses can be used to obtain arbitrary single-qubit rotations via the excitation of an intermediate trion state. Our theory takes into account a finite in-plane hole g-factor and hole mixing. We show that such rotations can be performed to high fidelities with pulses lasting a few tens of picoseconds.

(Some figures in this article are in colour only in the electronic version)

The spin of an electron in a quantum dot (QD) is currently viewed as one of the leading contenders in the drive to develop a qubit from which a practical quantum computer can be constructed [1]. The optical manipulation of such spins is an area of tremendous contemporary interest and offers a route to the ultra-fast control of qubits required to implement quantum logic.

In this paper we provide a theory of optically controlled single-qubit rotations of a single electron spin in a self-assembled InAs QD. Such dots are at the forefront of experimental effort in this field, and their promise for quantum computation purposes has been illustrated in a number of recent experiments. For example, the spin state of a QD electron has been prepared in a pure state to a very high fidelity [2], and the exchange interactions between optically excited coupled dots have recently been mapped [3].

The theory we describe here shows that by using two pulsed, Raman-detuned lasers one can obtain arbitrary single-qubit rotations of the electron spin. The rotation proceeds through the virtual excitation of an intermediate trion state. We show that, for realistic parameters, this can be accomplished with laser pulses lasting just a few tens of picoseconds. This is significantly faster than the decoherence time of the electron spin in such a dot, which has been measured at a few microseconds [4]. Furthermore, since the trion state is only virtually excited, this technique avoids problems arising from spontaneous emission from this state.

This work is an extension of that of Chen *et al* [5], who have presented a theory of single-qubit rotations for dots in which the heavy holes have a zero in-plane g-factor. Subsequent experiments [6] have shown that, although this is the case in GaAs fluctuation dots [7], the

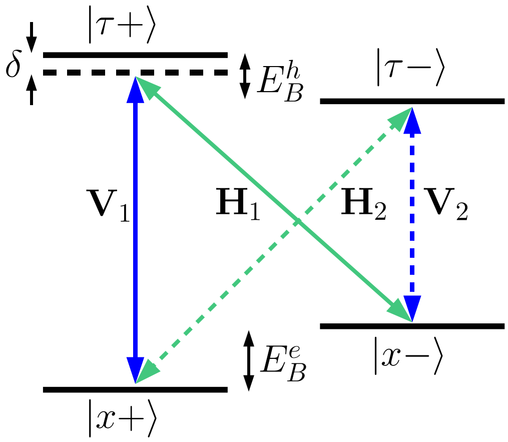

Figure 1. The four-level model of the electron–trion system in the Voigt basis consists of two Zeeman-split single-electron ground states $|x\pm\rangle$ with spins aligned in the x-direction, and two trion levels $|\tau\pm\rangle$ with heavy-hole spins also in the x-direction. The ground states are split by Zeeman energy $E_{\mathrm{B}}^{\mathrm{e}}$ and the trion states by $E_{\mathrm{B}}^{\mathrm{h}}$. Arrows indicate allowed optical transitions with $\mathbf{H}$ and $\mathbf{V}$ denoting two orthogonal linear polarizations. To obtain single-qubit rotations we pump transitions $\mathbf{V}_1$ and $\mathbf{H}_1$ with a common Raman detuning of δ.

g-factor in self-assembled InAs dots is actually finite. This is important because an in-plane magnetic field is vital to the rotation mechanism. Furthermore, in InAs dots we expect significant hole mixing, and this also needs to be taken into account. Finally, the analytic technique that we use here is somewhat different from that employed in [5], and is capable of describing the qubit rotations to a much higher fidelity.

1. Spin-trion qubit system

We consider a singly charged self-assembled InAs QD with growth direction z. The spin of the electron trapped in the QD is our qubit degree of freedom and we will perform rotations of this spin through the virtual excitation of an exciton within the dot. Figure 1 shows the four-level model that describes the pertinent features of this system. We apply a magnetic field in the x-direction. The Zeeman energy of a QD electron in this field is $\mathcal{H}_{\mathrm{B}}^{\mathrm{e}} = g_x^{\mathrm{e}}\mu_{\mathrm{B}}B_x s_x^{\mathrm{e}} \equiv E_{\mathrm{B}}^{\mathrm{e}} s_x^{\mathrm{e}}$, where g_x^{e} is the electronic g-factor, μ_{B} is the Bohr magneton, B_x is the magnitude of the field, and $s_x^{\mathrm{e}} = \pm 1/2$ corresponds to the electron spin.

The heavy-hole component of the trion also splits under this field in InAs QDs [6, 8], and can be described with a Zeeman Hamiltonian $\mathcal{H}_{\mathrm{B}}^{\mathrm{h}} = -g_x^{\mathrm{h}}\mu_{\mathrm{B}}B_x s_x^{\mathrm{h}} \equiv E_{\mathrm{B}}^{\mathrm{h}} s_x^{\mathrm{h}}$, where $s_x^{\mathrm{h}} = \pm 1/2$ are the eigenvalues of a pseudo-spin, the components of which correspond to heavy-hole states aligned in the x-direction, and g_x^{h} is the hole g-factor. Recent measurements have given the magnitudes of these g-factors as $|g_x^{\mathrm{e}}| = 0.46$ and $|g_x^{\mathrm{h}}| = 0.29$ [6]. For concreteness, we follow [8] here and assume that both these g-factors are negative, but this is in no way essential. In our initial treatment we will neglect hole mixing and show later that it can be incorporated into the analysis through a set of only very minor modifications.

The four levels of our model are then: the two electron ground states with spins in the x-direction, $|x\pm\rangle \equiv 2^{-1/2}\,(|\downarrow\rangle \pm |\uparrow\rangle)$, where $|\downarrow\rangle$ and $|\uparrow\rangle$ represent electron spins in the z-direction; and the two trion levels, $|\tau\pm\rangle \equiv 2^{-1}\,(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)\,(|\Downarrow\rangle \pm |\Uparrow\rangle)$, where $|\Downarrow\rangle = \left|\frac{3}{2}, -\frac{3}{2}\right\rangle$ and $|\Uparrow\rangle = \left|\frac{3}{2}, \frac{3}{2}\right\rangle$ denote heavy-hole states also aligned in the z-direction.

Figure 1 shows the allowed optical transitions between these levels. Due to the splitting of the trion level, these transitions are linearly polarized. We have defined the polarization vectors in terms of $\boldsymbol{\sigma}_\pm$ circular polarizations as $\mathbf{V} = 2^{-1/2}(\boldsymbol{\sigma}_- + \boldsymbol{\sigma}_+)$ and $\mathbf{H} = 2^{-1/2}(\boldsymbol{\sigma}_- - \boldsymbol{\sigma}_+)$ [8].

To obtain our qubit rotations, we illuminate the system with two phase-locked laser pulses propagating in the z-direction. We use one $\mathbf{V}$-polarized pulse with frequency ω_V and time-dependent Rabi frequency $\Omega_\mathrm{V}(t)$, and one $\mathbf{H}$-polarized pulse with frequency ω_H and Rabi frequency $\Omega_\mathrm{H}(t)$. With these lasers we pump the two transitions to the trion level $|\tau+\rangle$, labelled $\mathbf{V}_1$ and $\mathbf{H}_1$ in figure 1, with a common Raman detuning of δ. The model Hamiltonian in the basis $\{|x+\rangle, |x-\rangle, |\tau+\rangle, |\tau-\rangle\}$ is then

$$\mathcal{H} = \begin{pmatrix} +E_\mathrm{B}^\mathrm{e}/2 & 0 & \Omega_\mathrm{V}^* \mathrm{e}^{\mathrm{i}\omega_\mathrm{V} t + \mathrm{i}\alpha} & \Omega_\mathrm{H}^* \mathrm{e}^{\mathrm{i}\omega_\mathrm{H} t} \\ 0 & -E_\mathrm{B}^\mathrm{e}/2 & \Omega_\mathrm{H}^* \mathrm{e}^{\mathrm{i}\omega_\mathrm{H} t} & \Omega_\mathrm{V}^* \mathrm{e}^{\mathrm{i}\omega_\mathrm{V} t + \mathrm{i}\alpha} \\ \Omega_\mathrm{V} \mathrm{e}^{-\mathrm{i}\omega_\mathrm{V} t - \mathrm{i}\alpha} & \Omega_\mathrm{H} \mathrm{e}^{-\mathrm{i}\omega_\mathrm{H} t} & E_\tau + E_\mathrm{B}^\mathrm{h}/2 & 0 \\ \Omega_\mathrm{H} \mathrm{e}^{-\mathrm{i}\omega_\mathrm{H} t} & \Omega_\mathrm{V} \mathrm{e}^{-\mathrm{i}\omega_\mathrm{V} t - \mathrm{i}\alpha} & 0 & E_\tau - E_\mathrm{B}^\mathrm{h}/2 \end{pmatrix}, \tag{1}$$

where E_τ is the trion energy, α is the relative phase of the two lasers, and we have set $\hbar = 1$. The time dependence of Ω_V and Ω_H is understood.

We set the frequencies of the two lasers as $\omega_\mathrm{V} = E_\tau - \frac{1}{2}\Sigma_\mathrm{B} - \delta$ and $\omega_\mathrm{H} = E_\tau + \frac{1}{2}\Delta_\mathrm{B} - \delta$, where we have introduced

$$\begin{aligned} \Sigma_\mathrm{B} &= (g_x^\mathrm{e} + g_x^\mathrm{h})\mu_\mathrm{B} B_x = E_\mathrm{B}^\mathrm{e} - E_\mathrm{B}^\mathrm{h} \\ \Delta_\mathrm{B} &= (g_x^\mathrm{e} - g_x^\mathrm{h})\mu_\mathrm{B} B_x = E_\mathrm{B}^\mathrm{e} + E_\mathrm{B}^\mathrm{h}. \end{aligned} \tag{2}$$

Transforming to a rotating frame, we obtain the Hamiltonian

$$\mathcal{H} = \begin{pmatrix} 0 & 0 & \Omega_\mathrm{V}^* \mathrm{e}^{\mathrm{i}\alpha} & \Omega_\mathrm{H}^* \mathrm{e}^{\mathrm{i}\Delta_\mathrm{B} t} \\ 0 & 0 & \Omega_\mathrm{H}^* & \Omega_\mathrm{V}^* \mathrm{e}^{-\mathrm{i}\Sigma_\mathrm{B} t + \mathrm{i}\alpha} \\ \Omega_\mathrm{V} \mathrm{e}^{-\mathrm{i}\alpha} & \Omega_\mathrm{H} & \delta & 0 \\ \Omega_\mathrm{H} \mathrm{e}^{-\mathrm{i}\Delta_\mathrm{H} t} & \Omega_\mathrm{V} \mathrm{e}^{\mathrm{i}\Sigma_\mathrm{B} t - \mathrm{i}\alpha} & 0 & \delta \end{pmatrix}. \tag{3}$$

The terms oscillating with frequencies Δ_B and Σ_B describe the off-resonant driving of transitions $\mathbf{H}_2$ and $\mathbf{V}_2$ respectively, both of which involve the state $|\tau-\rangle$. In deriving our analytic solution for the behaviour of this system, it is these off-resonant transitions that we treat approximately, and it is therefore the magnitude of the quantities Δ_B and Σ_B that determines the accuracy of our description.

2. Adiabatic elimination of trion levels

We now seek an compact, approximate description of the evolution of the qubit sector and we shall proceed through the adiabatic elimination of the trion levels. This is appropriate for the situation we consider in which the detuning is sufficient that we only virtually occupy the trion level. The equations of motion for the wave function coefficients under the action of $\mathcal{H}$ are

$$\begin{aligned} \mathrm{i}\dot{c}_{x+} &= \Omega_\mathrm{V}^* \mathrm{e}^{\mathrm{i}\alpha} c_{\tau+} + \Omega_\mathrm{H}^* \mathrm{e}^{\mathrm{i}\Delta_\mathrm{B} t} c_{\tau-} \\ \mathrm{i}\dot{c}_{x-} &= \Omega_\mathrm{H}^* c_{\tau+} + \Omega_\mathrm{V}^* \mathrm{e}^{-\mathrm{i}\Sigma_\mathrm{B} t + \mathrm{i}\alpha} c_{\tau-} \\ \mathrm{i}\dot{c}_{\tau+} &= \delta c_{\tau+} + \Omega_\mathrm{V} \mathrm{e}^{-\mathrm{i}\alpha} c_{x+} + \Omega_\mathrm{H} c_{x-} \\ \mathrm{i}\dot{c}_{\tau-} &= \delta c_{\tau-} + \Omega_\mathrm{H} \mathrm{e}^{-\mathrm{i}\Delta_\mathrm{B} t} c_{x+} + \Omega_\mathrm{V} \mathrm{e}^{\mathrm{i}\Sigma_\mathrm{B} t - \mathrm{i}\alpha} c_{x-}. \end{aligned} \tag{4}$$

The equations for $c_{\tau\pm}$ can be rewritten as

$$\begin{aligned} \frac{\partial}{\partial t}\left(c_{\tau+}\mathrm{e}^{\mathrm{i}\delta t}\right) &= -\mathrm{i}\left\{\Omega_\mathrm{V} \mathrm{e}^{-\mathrm{i}\alpha} c_{x+} + \Omega_\mathrm{H} c_{x-}\right\} \mathrm{e}^{\mathrm{i}\delta t} \\ \frac{\partial}{\partial t}\left(c_{\tau-}\mathrm{e}^{\mathrm{i}\delta t}\right) &= -\mathrm{i}\left\{\Omega_\mathrm{H} \mathrm{e}^{-\mathrm{i}\Delta_\mathrm{B} t} c_{x+} + \Omega_\mathrm{V} \mathrm{e}^{\mathrm{i}\Sigma_\mathrm{B} t - \mathrm{i}\alpha} c_{x-}\right\} \mathrm{e}^{\mathrm{i}\delta t}. \end{aligned} \tag{5}$$

We proceed by integrating by parts and assuming that the envelopes $\Omega_{V,H}$ and thereby $c_{x\pm}$ are slowly varying functions of time. We thereby arrive at an approximate expression for the trion amplitudes in terms of the ground-state coefficients:

$$\begin{aligned} c_{\tau+} &\approx -\frac{\Omega_{\mathrm{V}} \mathrm{e}^{-\mathrm{i}\alpha}}{\delta} c_{x+} - \frac{\Omega_{\mathrm{H}}}{\delta} c_{x-} \\ c_{\tau-} &\approx -\frac{\Omega_{\mathrm{H}} \mathrm{e}^{-\mathrm{i}\Delta_{\mathrm{B}} t}}{\delta - \Delta_{\mathrm{B}}} c_{x+} - \frac{\Omega_{\mathrm{V}} \mathrm{e}^{\mathrm{i}\Sigma_{\mathrm{B}} t - \mathrm{i}\alpha}}{\delta + \Sigma_{\mathrm{B}}} c_{x-}. \end{aligned} \tag{6}$$

The validity of these expressions are conditioned on the following adiabatic constraints:

$$\begin{aligned} |\Omega_{\mathrm{V}} c_{x+}| &\gg \left| \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\Omega_{\mathrm{V}} c_{x+}}{\delta} \right) \right|, \\ |\Omega_{\mathrm{H}} c_{x-}| &\gg \left| \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\Omega_{\mathrm{H}} c_{x-}}{\delta} \right) \right|, \\ |\Omega_{\mathrm{H}} c_{x+}| &\gg \left| \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\Omega_{\mathrm{H}} c_{x+}}{\delta - \Delta_{\mathrm{B}}} \right) \right|, \\ |\Omega_{\mathrm{V}} c_{x-}| &\gg \left| \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\Omega_{\mathrm{V}} c_{x-}}{\delta + \Sigma_{\mathrm{B}}} \right) \right|, \end{aligned} \tag{7}$$

which mean that, as well as the laser amplitudes being slowly varying, the laser frequencies can not be too close to resonance with any the transitions.

Assuming that these conditions are met, we can substitute $c_{\tau\pm}$ from equation (6) into the remaining equations of motion (4), and obtain a closed set of equations of motion for the qubit coefficient $c_{x\pm}$:

$$\mathrm{i} \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} c_{x+} \\ c_{x-} \end{pmatrix} = h(t) \begin{pmatrix} c_{x+} \\ c_{x-} \end{pmatrix}, \tag{8}$$

where $h(t)$ is the effective Hamiltonian acting solely on the qubit space:

$$h(t) = -\begin{pmatrix} \frac{|\Omega_{\mathrm{V}}|^2}{\delta} + \frac{|\Omega_{\mathrm{H}}|^2}{\delta - \Delta_{\mathrm{B}}} & \frac{\Omega_{\mathrm{V}}^* \Omega_{\mathrm{H}}}{\delta} \mathrm{e}^{\mathrm{i}\alpha} \\ \frac{\Omega_{\mathrm{V}} \Omega_{\mathrm{H}}^*}{\delta} \mathrm{e}^{-\mathrm{i}\alpha} & \frac{|\Omega_{\mathrm{H}}|^2}{\delta} + \frac{|\Omega_{\mathrm{V}}|^2}{\delta + \Sigma_{\mathrm{B}}} \end{pmatrix}. \tag{9}$$

In writing this Hamiltonian, we have neglected further terms oscillating with frequency $2E_{\mathrm{B}}^{\mathrm{e}}$. These terms are expected to be negligible since, without them, the Rabi frequency of the effective model is $\Omega_{\mathrm{V}}^* \Omega_{\mathrm{H}}/\delta$, and thus provided that

$$2|E_{\mathrm{B}}^{\mathrm{e}}| \gg |\Omega_{\mathrm{V}}^* \Omega_{\mathrm{H}}/\delta|, \tag{10}$$

these terms are rapidly oscillating in comparison and thus approximately self-average to zero. This effective two-level Hamiltonian $h(t)$ forms the basis of our approach and provides a good account of the system, as we will show.

3. Time-evolution operator

Given that the qubit evolves under the action of effective Hamiltonian $h(t)$, we may once again make use of the adiabaticity of the system to obtain an approximate form for $U(t)$, the time-evolution operator of the system. First, let us specify that the envelopes $\Omega_{\mathrm{V}}(t)$ and $\Omega_{\mathrm{H}}(t)$ have the same shape but different amplitudes and write $\Omega_{\mathrm{H}}(t) = \nu \Omega_{\mathrm{V}}(t)$, where ν is constant in time. We can then write $h(t)$ in the form

$$h(t) = \tfrac{1}{2} \lambda(t) \boldsymbol{n} \cdot \boldsymbol{\sigma} + \mu \mathbb{I} \tag{11}$$

and can drop the constant term $\mu \mathbb{I}$. In general, both the forefactor λ and the unit vector $\boldsymbol{n}$ depend on time, but since we are assuming the same shape for both pulses, the axis $\mathbf{n}$ remains fixed, and all the time dependence is contained in $\lambda(t)$. We can then approximate the time-evolution operator as

$$U(t) = \exp\{-\mathrm{i}/2\Lambda(t)\, \boldsymbol{n}\cdot\boldsymbol{\sigma}\}, \tag{12}$$

where $\Lambda(t) = \int_{-\infty}^{t} \mathrm{d}t'\lambda(t')$. The final output gate operator is $U = U(t\to\infty)$.

The operator U represents an arbitrary single-qubit rotation. The rotation angle at time t is given by

$$\Lambda(t) = \frac{r}{|\delta(\Delta_\mathrm{B}-\delta)(\delta+\Sigma_\mathrm{B})|}\int_{-\infty}^{t}\Omega^2(t')\,\mathrm{d}t', \tag{13}$$

where

$$r^2 = \Sigma_\mathrm{B}^2(\Delta_\mathrm{B}-\delta)^2 + \Delta_\mathrm{B}^2(\delta+\Sigma_\mathrm{B})^2\nu^4 + 2(\Delta_\mathrm{B}-\delta)(\delta+\Sigma_\mathrm{B})\left[\Delta_\mathrm{B}(\Sigma_\mathrm{B}+2\delta)\nu^2 - 2\delta(\delta+\Sigma_\mathrm{B})\right]\nu^2. \tag{14}$$

Note that this expression for $\Lambda(t)$ depends only on the 'area under the pulse squared', and not on the details of the shape of the pulse. This is typical of the adiabatic approach we are pursuing here. For simplicity, we will consider a Gaussian pulse envelope, $\Omega_\mathrm{V} = A\exp\left(-t^2/2T^2\right)$, in which case the total angle is

$$\Lambda = \Lambda(t\to\infty) = \frac{rA^2\sqrt{\pi}T}{\delta(\Delta_\mathrm{B}-\delta)(\delta+\Sigma_\mathrm{B})}. \tag{15}$$

The axis of rotation has components

$$\begin{aligned} n_1 &= \cos\beta\cos\alpha \\ n_2 &= -\cos\beta\sin\alpha \\ n_3 &= \sin\beta \end{aligned} \tag{16}$$

where

$$\tan\beta = \frac{\Sigma_\mathrm{B}(\delta-\Delta_\mathrm{B}) + \Delta_\mathrm{B}(\delta+\Sigma_\mathrm{B})\nu^2}{2\nu(\delta-\Delta_\mathrm{B})(\delta+\Sigma_\mathrm{B})} \tag{17}$$

and α is the phase difference between the pulses.

Let us now describe two specific rotations as illustrative examples. We consider first a rotation about the **3** axis, which corresponds to a change in the relative phase of the qubit states. Only a single laser pulse is necessary and thus we set $\nu = 0$. The rotation is then precisely about the **3** axis, $\boldsymbol{n} = (0, 0, 1)$, by an angle of

$$\Lambda = \frac{\sqrt{\pi}A^2\Sigma_\mathrm{B}T}{\delta(\delta+\Sigma_\mathrm{B})}. \tag{18}$$

A rotation with the axis in the **1–2** plane includes a transfer of population between the two qubit levels. This requires two lasers and, by setting the relative amplitude of the two pulses according to

$$\nu = \sqrt{\frac{\Sigma_\mathrm{B}(\Delta_\mathrm{B}-\delta)}{\Delta_\mathrm{B}(\Sigma_\mathrm{B}+\delta)}}, \tag{19}$$

we set $n_3 = 0$, and obtain a rotation axis exactly in the **1–2** plane. The direction of this axis is then specified by the relative phase of the two lasers, α. In order for ν to be real and finite we require that $\Delta_\mathrm{B} \leqslant \delta < -\Sigma_\mathrm{B}$ for $\Delta_\mathrm{B}, \Sigma_\mathrm{B} < 0$, as is the case here.

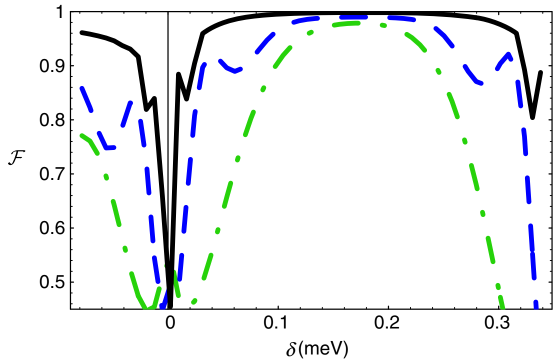

Figure 2. The fidelity $\mathcal{F}$, which reflects the degree to which our description approximates the exact evolution of the system, as a function of Raman detuning δ. Here we consider a π-rotation about the **3** axis, corresponding to a change in relative phase between the two qubit levels, using pulses of durations $T = 5$ ps (green dash–dot), 15 ps (blue dash) and 50 ps (black solid line). As is clear, for this type of rotation, excellent agreement ($\mathcal{F}$ close to unity) can be obtained with even very short pulses ($T \approx 5$ ps). The model parameters are $g_x^{\rm e} = -0.46$ and $g_x^{\rm h} = -0.29$ with a magnetic field of 8 T. Laser parameters were $\nu = 0$, with A chosen to give $\Lambda = \pi$.

4. Fidelity

The preceding sections demonstrate that arbitrary single-qubit rotations are possible within this set-up. The important question then arises as to how fast these operations can be performed. This requires that we know the degree to which our approximate description matches the actual behaviour of the system. This we quantify with the fidelity between the calculated operation and the result obtained by numerical integration of the Schrödinger equation. The fidelity is defined as [9]

$$\mathcal{F} = \overline{\langle \Psi_{\rm in} | \widetilde{U}^\dagger \rho_{\rm out} \widetilde{U} | \Psi_{\rm in} \rangle}, \tag{20}$$

where the overline represents an average over all input states $|\Psi_{\rm in}\rangle$, $\widetilde{U}$ is the predicted operation, and $\rho_{\rm out}$ is the actual output density matrix. In evaluating the fidelity from our numerics, we have used the method described in [10].

We calculate the fidelities for the two $\Lambda = \pi$ rotations above: one about the **3** axis and the other about the **1** axis. In figure 2 we plot $\mathcal{F}$ as function of the laser detuning δ for the rotation about the **3** axis for several values of the pulse duration T. This figure shows that, for the **3**-axis rotation, even very short pulses ($T \approx 5$ ps) can be used with fidelity close to unity provided that the detuning δ is chosen appropriately. In figure 3 we plot the same thing for the rotation about the **1** axis and observe that longer pulse durations are required to obtain high fidelities. Irregularities in $\mathcal{F}$ occur in both these figures at values of $\delta = 0, \Delta_{\rm B}, -\Sigma_{\rm B}$. These are the regions at which we know from equation (7) that the perturbative theory breaks down.

We now examine the dependence on the applied magnetic field of the maximum obtainable fidelity for a given pulse duration. We consider a $\Lambda = \pi$, **1**-axis rotation, since this is the more demanding rotation, and in figure 4 plot the results for several different magnetic fields. As is clear, this field determines the pulse duration required to obtain a given fidelity. For the parameters used here a field of 8 T gives a fidelity of 95% for a pulse duration of about 50 ps.

The required pulse duration is reduced if the g-factors are larger. In particular, we have used here an electronic g-factor of $|g_x^{\rm e}| = 0.46$. This is quite small compared with other measurements reported for InAs QDs, which have yielded values of $|g_x^{\rm e}| = 0.6$ [2] and $|g_x^{\rm e}| \approx 0.9$ [11]. Use of such dots will significantly reduce either the magnetic field or the

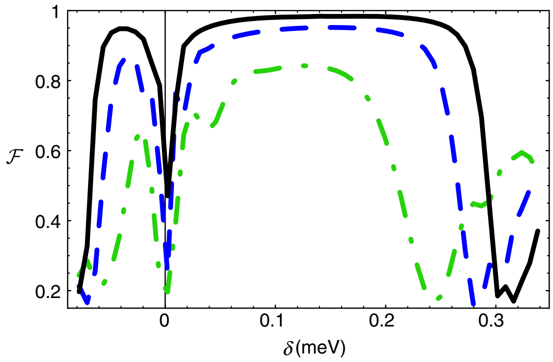

Figure 3. Same as figure 2, except that here the rotation is about the **1** axis and the pulse durations are $T = 20$ ps (green dash–dot), 50 ps (blue dashes) and 100 ps (black solid line). Longer times are required to perform this operation, which involves a transfer of population between qubit levels. Pulse parameters ν and A were chosen to give the desired rotation axis and angle, $\Lambda = \pi$, for each value of the detuning δ.

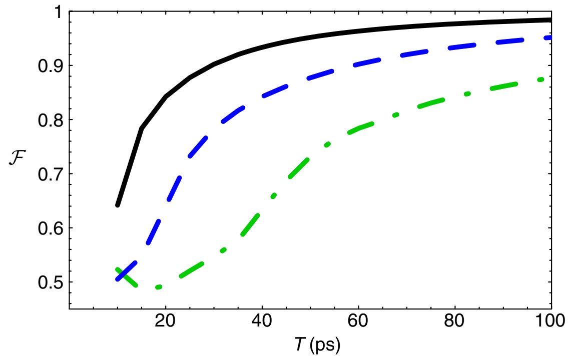

Figure 4. The maximum fidelity $\mathcal{F}$ for a $\Lambda = \pi$ rotation about the **1** axis as a function of pulse duration T. Results are shown for magnetic fields of 2 T (green (dash–dot)), 4 T (blue (dash)), and $B = 8$ T (black (solid line)), with g-factors as in figure 2. Fidelity increases with both magnetic field and pulse duration. For $B = 8$ T a fidelity of 95% can be obtained with pulse durations of ≈50 ps.

pulse duration required to perform high-fidelity operations. It should also be noted that a π-rotation about the **1** axis is the worst case example and that all other rotations can be achieved in less time.

In any case, these time scales are much shorter than the trion lifetime in InAs dots, which has been measured to be 1 ns or longer [12, 13]. This, coupled with the fact that the gate operation proceeds essentially adiabatically, means that the effects of spontaneous emission from the trion has negligible effect on the gate operation.

5. Hole mixing

We now include the effects of hole mixing in our analysis. The dominant mixing terms in the Luttinger Hamiltonian [14, 15] mean that, rather than the 'bare' heavy-hole states $\left|\frac{3}{2}, \pm\frac{3}{2}\right\rangle$, the actual states are better approximated as

$$\left|H_z^{\pm}\right\rangle = \cos\theta_m \left|\tfrac{3}{2}, \pm\tfrac{3}{2}\right\rangle - \sin\theta_m \mathrm{e}^{\mp \mathrm{i}\phi_m} \left|\tfrac{3}{2}, \mp\tfrac{1}{2}\right\rangle, \tag{21}$$

where $\left|\frac{3}{2}, \mp\frac{1}{2}\right\rangle$ are light-hole states, θ_m and ϕ_m are mixing angles, and we have used the growth direction as the quantization axis.

In terms of orbital and spin degrees of freedom, the valence-band electron states are [15][1]

$$\begin{aligned} \left|\tfrac{3}{2}, \pm\tfrac{3}{2}\right\rangle_e &= \left|\pm 1, \pm\tfrac{1}{2}\right\rangle_e \\ \left|\tfrac{3}{2}, \pm\tfrac{1}{2}\right\rangle_e &= \sqrt{\tfrac{1}{3}} \left|\pm 1, \mp\tfrac{1}{2}\right\rangle_e \pm \sqrt{\tfrac{2}{3}} \left|0, \pm\tfrac{1}{2}\right\rangle_e, \end{aligned} \tag{22}$$

where, on the right-hand side, the first index corresponds to the orbital degree of freedom,

$$|\pm 1\rangle = \sqrt{1/2}\,(|X\rangle \pm \mathrm{i}\,|Y\rangle); \qquad |0\rangle = |Z\rangle, \tag{23}$$

and the second to the electron spin $\pm 1/2$. The make-up of these states means that, for light propagating in the z-direction, transitions involving the light-hole components of $\left|H_z^{\pm}\right\rangle$ are possible and that they acquire an extra factor of $\sqrt{1/3}$ in the matrix element.

We are now in a position to consider the Hamiltonian of the system including hole mixing, for which we use the same basis as for equation (3), except that now we use the hole states $\left|H_x^{\pm}\right\rangle = 1/\sqrt{2}\left(\left|H_z^{-}\right\rangle \pm \left|H_z^{+}\right\rangle\right)$. If we illuminate the dot with the same linear polarizations as before, hole mixing means that all four transitions are driven by each polarization and the Hamiltonian of the system is far more complicated than that considered in the foregoing analysis. However, by adjusting the polarizations of the two lasers, we can reach a situation where the Hamiltonian is identical to that of equation (3), but with renormalized parameters. The single-qubit rotations then follow directly.

The two laser polarizations that accomplish this are $\mathbf{V}' = 2^{-1/2}(\boldsymbol{\sigma}_+ + \mathrm{e}^{\mathrm{i}\mu_+}\boldsymbol{\sigma}_-)$ and $\mathbf{H}' = 2^{-1/2}(\boldsymbol{\sigma}_+ - \mathrm{e}^{-\mathrm{i}\mu_-}\boldsymbol{\sigma}_-)$ with the phases

$$\mathrm{e}^{\mathrm{i}\mu_\pm} = \frac{\sqrt{3}\cos\theta_m \pm \sin\theta_m \mathrm{e}^{\pm\mathrm{i}\phi_m}}{\sqrt{3}\cos\theta_m \pm \sin\theta_m \mathrm{e}^{\mp\mathrm{i}\phi_m}}. \tag{24}$$

Note that these two polarization are non-orthogonal in general. Using these polarizations we obtain a Hamiltonian the same form as equation (3), but with Ω_{V} and Ω_{H} being replaced by

$$\widetilde{\Omega}_{V'} = \Omega_{V'} \frac{1 + 2\cos 2\theta_m}{3\cos\theta_m + \sqrt{3}\mathrm{e}^{-\mathrm{i}\phi_m}\sin\theta_m}, \qquad \widetilde{\Omega}_{H'} = \Omega_{H'} \frac{1 + 2\cos 2\theta_m}{3\cos\theta_m - \sqrt{3}\mathrm{e}^{\mathrm{i}\phi_m}\sin\theta_m}, \tag{25}$$

where $\Omega_{V'}$ and $\Omega_{H'}$ are the actual Rabi frequencies of the two transitions. Use of these new polarizations therefore allows us to circumvent the effects of hole mixing and proceed directly as outlined in the previous sections.

6. GaAs dots

We now describe briefly the application of the above theory to QDs in which the trion level does not split under in-plane magnetic field. This was the situation studied in [5], but the approach we pursue here is different and leads to an improved description of this system.

With the in-plane hole g-factor equal to zero, and with no hole mixing, illumination with $\boldsymbol{\sigma}_+$-circularly polarized light propagating in the growth direction excites a trion state $|\tau_z+\rangle$ containing a spin-up heavy hole $|\Uparrow\rangle$ aligned in the growth direction. We employ two such $\boldsymbol{\sigma}_+$ lasers set to a detuning δ from the Raman transition between the two electron

[1] We have used different global phases for some of the wave functions as compared with [15].

ground states $|x\pm\rangle$. A full description is given in [5], where it is shown that in the basis $\{|x+\rangle, |x-\rangle, |\tau_z+\rangle, \}$, the Hamiltonian for the system in the rotating frame is given by

$$\mathcal{H} = \begin{pmatrix} 0 & 0 & \Omega_\uparrow^* e^{i\alpha} + \Omega_\downarrow^* e^{iE_B^e t} \\ 0 & 0 & \Omega_\uparrow^* e^{-iE_B^e t + i\alpha} + \Omega_\downarrow^* \\ \Omega_\uparrow e^{-i\alpha} + \Omega_\downarrow e^{-iE_B^e t} & \Omega_\uparrow e^{iE_B^e t - i\alpha} + \Omega_\downarrow & \delta \end{pmatrix}, \tag{26}$$

where $\Omega_\uparrow(t)$ and $\Omega_\downarrow(t)$ are the Rabi frequencies of the two $\boldsymbol{\sigma}_+$-induced transitions, and E_B^e is the Zeeman splitting of the electron.

From this Hamiltonian we can derive an effective two-level model for the qubit sector exactly as in the previous sections. We obtain

$$h_1(t) = -\begin{pmatrix} \frac{|\Omega_\uparrow|^2}{\delta} + \frac{|\Omega_\downarrow|^2}{\delta - E_B^e} & \frac{\Omega_\uparrow^* \Omega_\downarrow e^{i\alpha}}{\delta} \\ \frac{\Omega_\uparrow \Omega_\downarrow^* e^{-i\alpha}}{\delta} & \frac{|\Omega_\uparrow|^2}{\delta + E_B^e} + \frac{|\Omega_\downarrow|^2}{\delta} \end{pmatrix}. \tag{27}$$

This has the same form as equation (9) with the substitutions $\Omega_V \to \Omega_\uparrow$, $\Omega_H \to \Omega_\downarrow$ and $\Sigma_B, \Delta_B \to E_B^e$. The axis and angle of the corresponding single-qubit rotations follow directly.

Although derived in a different way, the work of Chen *et al* [5] essentially posits that the qubit is governed by the effective Hamiltonian

$$h_2(t) = -\frac{1}{\delta}\begin{pmatrix} |\Omega_\uparrow|^2 & \Omega_\uparrow^* \Omega_\downarrow e^{i\alpha} \\ \Omega_\uparrow \Omega_\downarrow^* e^{-i\alpha} & |\Omega_\downarrow|^2 \end{pmatrix}. \tag{28}$$

This differs from the Hamiltonian h_1 by the absence of the on-diagonal terms with $\delta \pm E_B^e$ in the denominator that describe the off-resonant effects of the qubit levels being driven by both lasers. These terms have a significant impact on the accuracy of the description with pulses as short as we use here. With parameters for which the description based on $h_1(t)$ has a fidelity of 95%, that based on $h_2(t)$ will typically have a fidelity of the order of 75%. This shows that the treatment of these off-resonant terms is essential to the proper understanding of single-qubit rotation in QDs and that the adiabatic elimination approach pursued here provides just such a treatment.

7. Summary

We have presented a theory of single-qubit rotations in InAs quantum dots. This theory provides a description of the behaviour of the system to high fidelity for realistic dot parameters and short laser pulses.

Our theory shows how two Raman-detuned lasers can be used to obtain arbitrary single-qubit rotations, and that this can be accomplished with laser pulses of duration lasting a few tens of picoseconds. We have also shown that hole mixing can be simply incorporated into this scheme through a change in laser polarizations.

This work differs from that presented in [5] not only in that it applies to systems in which the hole has an in-plane g-factor but also in the methodology. The adiabatic elimination approach that we have utilized here provides a better account of the system because it includes the effects of the unwanted off-resonant transitions.

Acknowledgment

This work was supported by ARO/NSA-LPS.

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